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Entrainment in Rotary Cylinders

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To design a rotary cylinder, knowledge of the passage or holdup of solid and the heat transfer in the cylinder is required. However, an optimum design usually cannot be made without an understanding of the entrainment occurring inside the cylinder. Very limited knowledge on the entrainment has been available in the literature. Friedman and Marshall (1949) studied the holdup and entrainment in a rotary dryer. They report some entrainment data and correlations but did not obtain an entrainment equation. An equation is presented by Khodorov (1961). The equation, however, does not include important design variables such as the rotation speed and slope of the cylinder and the solid feed rate. Therefore, the equation is not adequate for design applications. An entrainment equation relating to the design variables is developed in this study.

THEORETICAL CONCEPT

The path of solid in a rotary cylinder is such that a particle in the solid bed remains stationary with respect to the cylinder until it reaches a position close to the surface of the bed. From this point, it cascades down along the bed surface and comes to rest at some lower position. The cycle is then repeated (Saeman, 1951). The solid particles contacted with the cylinder wall will be carried up to a position slightly above the bed surface due to the friction between solid and wall. In case there are flights on the cylinder wall the carry-up will be pronounced. The carry-up gives the particles a better contact with the gas. Hence the entrainment at the higher edge of the bed surface, edge entrainment, should be more important than the entrainment occurring on the bed surface, surface entrainment. When the solid particles are cascading down on the surface, the finer particles filter down through the bed. This also causes the surface entrainment to be less significant.

The edge entrainment can be modeled by considering the solid material at the bottom layer of the bed as blocks which are carried up by the wall friction alone or with flights and then fall onto the bed surface as illustrated in Figure 1. In practical operations, the entrainment severity is maintained so low that the entrainment rate dW/dL may be assumed to be proportional to the falling rate of the blocks, the concentration of fines in the blocks C and the intensity of the eddy flow I_e. I_e is a function of the density, viscosity, and velocity of gas and the dimension of gas flow cross-sectional area. Noting that the block falling rate is proportional to the product of cylinder rotation speed N and the cylinder diameter D, we can write

$$\frac{dW}{dL} \propto ND\, CI_e \tag{1}$$

Since fines will filter through the larger solid particles, the fines will be concentrated in the bottom solid layer. Thus,

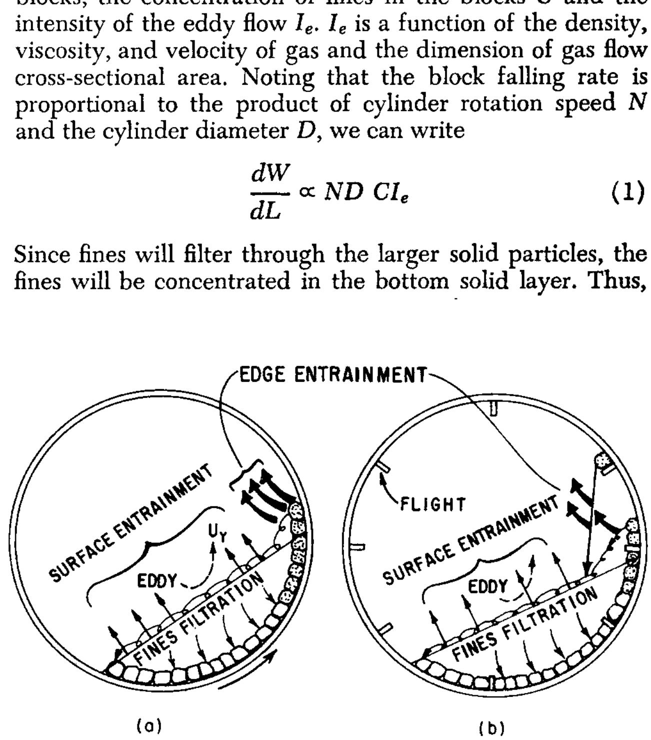

Fig. 1. Entrainment mechanism: (a) no flights, (b) with flights.

we may assume that most of the fines will settle in the bottom layer and that the following relation holds due to the conservation of material:

$$C = \frac{V}{Ah} C_0 = \frac{k}{h} \sqrt{V} C_0 \tag{2}$$

where k is equal to $\sqrt{V}/A$, that is, the ratio of the square root of bed cross section area to the bottom layer length. This ratio is a function of the fraction of cylinder volume occupied by solid bed X and is fairly constant in the range of practical operations. For example, the ratio varies from 2.89 to 2.75 when X varies from 0.105 to 0.156. The average fines concentration C_0 is a function of the gas eddy upward velocity U_Y, fines terminal velocity U_t, and solid size distribution factor ϕ, that is,

$$C_0 = f(\phi, U_t, U_Y) \tag{3}$$

Table 1. Effect of Feed Rate on Entrainment
(Air rate = 0.665 kg/s/m², cylinder diam. = 30.48 cm, slope = 0.0154 m/m, rotation speed = 1.047 rad/s, Wagner sand)

Solid feed rate, $F_1 \times 10^4$ ($m^3/s/m^2$)	Entrainment, W_p (wt. % of feed)	Entrainment, $W_1 \times 10^6$ $W_1 = F_1W_p/100$ ($m^3/hr/m^2$)
1.19*	5.7	6.78 } av. = 6.93
1.19	5.95	7.08
2.84	3.05	8.66 } av. = 9.65
2.84	3.45	9.80
2.84	3.7	10.5
4.15	2.7	11.2

* 1.19×10^{-4} m³/s/m² = 1.4 ft³/hr/ft².

Table 2. Effect of Feed Rate on Holdup
(Wagner sand)

Solid feed rate term, $F_2 \times 10^2$ ($m^2/s/m^2/(rad/s)^{0.9}/m$)	Bed volume holdup, V_1 (% of dryer volume)	Solid feed rate, $F_1 \times 10^4$ ($m^3/s/m^2$)
2.12*	3.6	1.03
4.23	6	2.07
6.35	8	3.11
8.47	9.4	4.15

$F_1 = F_2SN^{0.9}D$ where S = 0.0154 m/m, N = 1.047 rad/s, and D = 30.48 cm.
* 2.12×10^{-2} in S.I. units = 10 ft³/hr/ft²/RPM$^{0.9}$/ft.

DEVELOPMENT OF ENTRAINMENT EQUATION

Combining Equations (1), (2), and (3), we can obtain

$$\frac{dW}{dL} = KND\sqrt{V} f(\phi, U_t, U_Y) I_e \tag{4}$$

where K is a proportionality constant which will vary with the roughness between the solid and cylinder wall or the flight design, and $f(\phi, U_t, U_Y)I_e = f(\phi, U_x, d, \rho, \mu, D, X)$.

The theoretical entrainment equation, Equation (4) consists of a fines availability function $ND\sqrt{V}$ and a fluid dynamic function $f(\phi, U_t, U_Y)I_e$. The fluid dynamic function has been studied by Khodorov (1961). In this study the availability function was developed into an entrainment equation as follows.

The bed holdup V is a function of cylinder design and the operating conditions. This has been well investigated, and various types of holdup equations are available (for example, Kramers and Croockewit, 1952; Saeman, 1951; Pickering et al., 1951; Vahl and Kingma, 1952; Friedman and Marshall, 1949; Bayard, 1945). In this study the following general equation (see Friedman and Marshall, 1949) will be used to show that Equation (4) can be developed further:

$$V \propto \left[\frac{Ff(\theta)}{SDN^a}\right]^b \tag{5}$$

where $f(\theta)$ has been correlated as $\theta + 0.42$ or $\theta^{1/2}$, 'a' as 1 or 0.9, and 'b' as 1 in general. However, 'b' may vary from about 1.4 to 0.6 as expressed in Figure 7 of Friedman and Marshall (1949).

Inserting Equation (5) into Equation (4) and then integrating, we obtain the desired entrainment equation

$$W = K_1 ND \left[\frac{Ff(\theta)}{SDN^a}\right]^{b/2} \int_0^L f(\phi, U_t, U_Y) I_e$$

or

$$W = K_2 N^{1-ab/2} [FS^{-1} f(\theta)]^{b/2} = K_2 \sqrt{NFS^{-1}\theta^{1/2}} \tag{6}$$

where

$$K_2 = K_1 D^{1-b/2} \int_0^L f(\phi, U_t, U_Y) I_e = K_1 \sqrt{D} L [f(\phi, U_t, U_Y) I_e]_{av}.$$

EXPERIMENTAL DATA SUPPORT

The availability function $ND\sqrt{V}$ has the following experimental support:

The data shown in Tables 1, 2, and 3 are from Figures 15, 7, and 14 of (Friedman and Marshall, 1949), respectively. The entrainment and the holdup data in Tables 1 and 2 are plotted in Figure 2. The two linear lines in this

Table 3. Effect of Rotation on Entrainment
(Air rate = 0.570 kg/s/m², cylinder diameter = 30.48 cm, slope = 0.0154 m/m, Wagner sand)

Solid feed rate, $F_1 \times 10^4$ ($m^3/s/m^2$)	0.351 rad/s W_p	0.351 rad/s $W_1 \times 10^6$	1.047 rad/s W_p	1.047 rad/s $W_1 \times 10^6$	1.571 rad/s W_p	1.571 rad/s $W_1 \times 10^6$
(a) 1.19	3.7	4.40	5.9	7.02	6.9	8.21
(b) 2.84	1.7	4.83	3.05	8.66	—	—
$(b)^* = (b) \times \frac{6.93}{9.65}$	—	3.47	—	6.22	—	—

W_p = entrainment, wt % of solid feed.
$W_1 = F_1W_p/100$ = entrainment, ($m^3/s/m^2$).

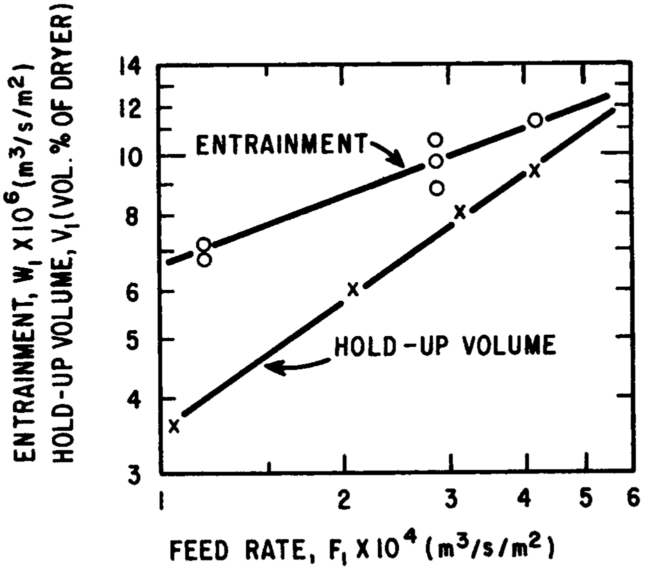

Fig. 2. Correlation of solid feed rate vs. entrainment and holdup volume.

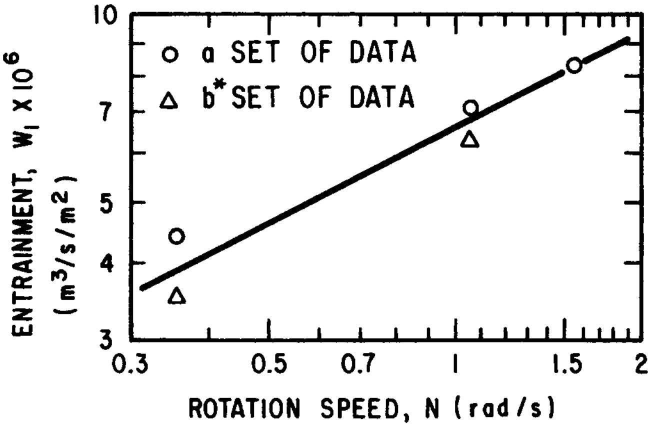

Fig. 3. Correlation of rotation speed vs. entrainment.

figure indicate

$$W_1 \propto F_1^{0.35} \quad \text{and} \quad V_1 \propto F_1^{0.7}$$

Since $W_1 = W/(\pi D^2/d/4)$, $V_1 = 100V/(\pi D^2/4)$, and d, S, N and D are fixed values here, we can have

$$W_1 \propto V_1^{0.5} \quad \text{and} \quad W \propto V^{0.5} \qquad (7)$$

This is to show the validity of Equation (4) with respect to the bed holdup V.

In order to find out the effect of rotation speed N on entrainment with a and b sets of data in Table 3, the b set of data was adjusted to obtain b^* set of data by multiplying with a factor, 6.93/9.65. This factor is obtained from Table 1 and is used to eliminate the effect due to the solid flow rate difference between a and b sets of data. Then a and b^* sets of data were plotted as shown in Figure 3. The straight line in this figure indicates

$$W_1 \propto N^{0.5} \qquad (8)$$

Since the rotation speed change will affect V, Equation (8) can be written as

$$W_1 \propto f(N)\, f(V) \propto N^{0.5} \qquad (9)$$

where $f(N)$ is the effect of rotation speed on entrainment while the other variables in Equation (4), including V, are not changing and $f(V)$ is that of V change induced by the rotation speed change. It is generally accepted that the bed holdup is inversely proportional to the rotation speed (Bayard, 1945; Kramers and Croockewit, 1952), that is

$$V \propto N^{-1} \qquad (10)$$

Knowing the effect of V on entrainment from Equation (7) and using Equation (10), we can write

$$f(V) = V^{0.5} \propto N^{-0.5} \qquad (11)$$

From Equations (9) and (11), we obtain

$$f(N) = N \qquad (12)$$

Equation (12) is to show the validity of Equation (4) with respect to the rotation speed.

DISCUSSION

The fines entrainment equation, Equation (6), is obtained by choosing $f(\theta) = \theta^{1/2}$ and $a = b = 1$. This equation should be good for general use. However, if a specific kind of Equation (5) is known to be more suitable for a certain application, it should be used with Equation (4) to form a better entrainment equation.

The surface entrainment is generally not important and ignored in the development of the entrainment equation. When the cylinder wall is very smooth and no flights are used, the surface entrainment may become significant. However, the result should not be greatly affected because the surface entrainment has a rate equation similar to that of the edge entrainment, Equation (4). The surface entrainment rate is proportional to (1) the rate of solid blocks passing on the solid bed surface, which is also proportional to the product of the rotation speed N and cylinder diameter D, (2) the bed top surface area which is approximately proportional to the square root of bed holdup volume $\sqrt{V}$, (3) the fines concentration C_0, and (4) the eddy intensity, I_e.

CONCLUSION

The entrainment equation, Equation (6), is useful for predicting the effects of the important variables: rotation speed, solid feed rate, cylinder slope, and angle of repose of the solid, which are not included in Khodorov's equation (1961). Equation (6) can be used together with Khodorov's equation for a more complete study on the operation and design of a rotary cylinder.

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NOTATION

- A = solid and cylinder contact surface area per unit cylinder length, m^2/m
- a, b = constants
- C = concentration of fines in the solid block lifted up by cylinder wall, kg/m^3
- C_0 = average concentration of fines in the solid, kg/m^3
- d = density of solid, kg/m^3
- D = inside diameter of cylinder, m
- F = feed rate of solid, kg/s
- F_1 = $F/d(\pi D^2/4)$, $m^3/s/m^2$

F_2 = $F_1/(SN^{0.9}D)$, feed rate term
$f(N)$ = effect of rotation speed on entrainment
$f(V)$ = effect of bed holdup on entrainment
h = thickness of bottom solid layer, m
I_e = intensity of eddy flow
K, K_1, K_2, k = proportionality constants
L = length of cylinder, m
N = cylinder rotation speed, rad/s
S = cylinder slope, m/m
U_t = average fines terminal velocity, m/s
U_x = gas velocity in the direction of cylinder axis, m/s
U_Y = eddy upward velocity, m/s
V = bed holdup volume per unit cylinder length, m^3/m
V_1 = $100\,V/(\pi D^2/4)$, holdup volume percentage
W = fines entrainment rate, kg/s
W_1 = $W/d(\pi D^2/4)$, $m^3/s/m^2$
X = fraction of cylinder volume occupied by solid bed
θ = dynamic angle of repose of solid, rad
ϕ = solid size distribution factor
ρ, μ = density and viscosity of gas, respectively

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Minimum Pressure in Dynamic Menisci

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The authors are interested in describing the properties of menisci, which are curved, free liquid-fluid interfaces in which surface tension effects are appreciable. This paper is concerned with the prediction of the location of the minimum pressure point in dynamic menisci, that is, in menisci distorted by flow. The geometry considered is free coating of flat sheets, as shown in Figure 1a. This geometry can be obtained in the laboratory using any one of several bath arrangements. One type of bath used to provide free coating is shown in diagram form in Lee and Tallmadge (1973b) and elsewhere.

The resultant profile of the liquid surface pressure P_s, as noted in Figure 1b, passes through a minimum at some point near the bath surface, often near the point where the interfacial curvature C is large. The normal stress boundary condition for this geometry is given by Batchelor (1967) as

$$\Delta P = P_0 - P_s = \sigma C + 2\mu\left(\frac{\partial u_s}{\partial s}\right) \tag{1}$$

Here

$$\sigma C \equiv \frac{\sigma}{R} \equiv \frac{\sigma\ \partial^2 h/\partial x^2}{[1 + (\partial h/\partial x)^2]^{3/2}} \tag{1a}$$

Here x is the vertical height. Based on Scriven (1960), it can be shown that Equation (1) applies where the following effects are negligible: surface viscosity and elasticity, surface tension gradients, and interphase mass transport; this is the case in studies with viscous oils by Lee and Tallmadge (1973a) and others.

The minimum pressure is of interest in order to characterize pressure profiles in dynamic menisci. The profiles themselves have been found useful in predicting the location of the free surface (Lee, 1973) and in calculating the relative contribution of the curvature and viscous terms in Equation (1) (Lee, 1973); furthermore, it is believed that a knowledge of profiles provide a better scientific description of menisci, stagnation points, and vortexes and may lead to a better understanding of nonuniformities in liquid

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